12-9 Binomial Experiments

Main Ideas

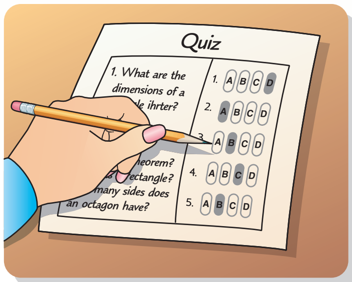
- Use binomial experiments to find probabilities.
- Find probabilities for binomial experiments.

New Vocabulary

binomial experiment

GET READY for the Lesson

What is the probability of getting exactly 4 questions correct on a 5-question multiple-choice quiz if you guess at every question?



Binomial Expansions You can use the Binomial Theorem to find probabilities in certain situations where there are two possible outcomes. The 5 possible ways of getting 4 questions right r and 1 question wrong w are shown at the right. This chart shows the combination of 5 things (answer choices) taken 4 at a time (right answers) or $C(5, 4)$.

w	r	r	r	r
r	w	r	r	r
r	r	w	r	r
r	r	r	w	r
r	r	r	r	w

The terms of the binomial expansion of $(r + w)^5$ can be used to find the probabilities of each combination of right and wrong.

$$(r + w)^5 = r^5 + 5r^4w + 10r^3w^2 + 10r^2w^3 + 5rw^4 + w^5$$

Coefficient	Term	Meaning
$C(5, 5) = 1$	r^5	1 way to get all 5 questions right
$C(5, 4) = 5$	$5r^4w$	5 ways to get 4 questions right and 1 question wrong
$C(5, 3) = 10$	$10r^3w^2$	10 ways to get 3 questions right and 2 questions wrong
$C(5, 2) = 10$	$10r^2w^3$	10 ways to get 2 questions right and 3 questions wrong
$C(5, 1) = 5$	$5rw^4$	5 ways to get 1 question right and 4 questions wrong
$C(5, 0) = 1$	w^5	1 way to get all 5 questions wrong

The probability of getting a question right that you guessed on is $\frac{1}{4}$. So, the probability of getting the question wrong is $\frac{3}{4}$. To find the probability of getting 4 questions right and 1 question wrong, substitute $\frac{1}{4}$ for r and $\frac{3}{4}$ for w in the term $5r^4w$.

$$P(\text{4 right, 1 wrong}) = 5r^4w$$

$$= 5\left(\frac{1}{4}\right)^4\left(\frac{3}{4}\right) \qquad r = \frac{1}{4}, w = \frac{3}{4}$$

$$= \frac{15}{1024} \qquad \text{Multiply.}$$

The probability of getting exactly 4 questions correct is $\frac{15}{1024}$ or about 1.5%.

Study Tip

Look Back

To review the **Binomial Theorem**, see Lesson 11-7.

EXAMPLE 1 Binomial Theorem

If a family has 4 children, what is the probability that they have 3 boys and 1 girl?

There are two possible outcomes for the gender of each of their children: boy or girl. The probability of a boy b is $\frac{1}{2}$, and the probability of a girl g is $\frac{1}{2}$.

$(b + g)^4 = b^4 + 4b^3g + 6b^2g^2 + 4bg^3 + g^4$

The term $4b^3g$ represents 3 boys and 1 girl.

$$P(3 \text{ boys, } 1 \text{ girl}) = 4b^3g$$

$$= 4\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right) \qquad b = \frac{1}{2}, g = \frac{1}{2}$$

$$= \frac{1}{4} \qquad \text{The probability is } \frac{1}{4} \text{ or } 25\%.$$

CHECK Your Progress

1. If a coin is flipped six times, what is the probability that the coin lands heads up four times and tails up two times?

Real-World Link

As of 2005, the National Hockey League record for most goals in a game by one player is seven. A player has scored five or more goals in a game 53 times in league history.

Source: NHL

Binomial Experiments Problems like Example 1 that can be solved using binomial expansion are called **binomial experiments**.

KEY CONCEPT — *Binomial Experiments*

A binomial experiment exists if and only if all of these conditions occur.

- There are exactly two possible outcomes for each trial.
- There is a fixed number of trials.
- The trials are independent.
- The probabilities for each trial are the same.

A binomial experiment is sometimes called a *Bernoulli experiment*.

EXAMPLE 2 Binomial Experiment

SPORTS Suppose that when hockey star Martin St. Louis takes a shot, he has a $\frac{1}{7}$ probability of scoring a goal. He takes 6 shots in a game.

a. What is the probability that he will score exactly 2 goals?

The probability that he scores on a given shot is $\frac{1}{7}$, and the probability that he does not is $\frac{6}{7}$. There are $C(6, 2)$ ways to choose the 2 shots that score.

$$P(2 \text{ goals}) = C(6, 2)\left(\frac{1}{7}\right)^2\left(\frac{6}{7}\right)^4 \qquad \text{If he scores on 2 shots, he fails to score on 4 shots.}$$

$$= \frac{6 \cdot 5}{2}\left(\frac{1}{7}\right)^2\left(\frac{6}{7}\right)^4 \qquad C(6, 2) = \frac{6!}{4!2!}$$

$$= \frac{19{,}440}{117{,}649} \qquad \text{Simplify.}$$

The probability of exactly 2 goals is $\frac{19{,}440}{117{,}649}$ or about 17%.

b. What is the probability that he will score at least 2 goals?

Instead of adding the probabilities of getting exactly 2, 3, 4, 5, and 6 goals, it is easier to subtract the probabilities of getting exactly 0 or 1 goal from 1.

$$\begin{aligned} P(\text{at least 2 goals}) &= 1 - P(0 \text{ goals}) - P(1 \text{ goal}) \\ &= 1 - C(6, 0)\left(\frac{1}{7}\right)^0\left(\frac{6}{7}\right)^6 - C(6, 1)\left(\frac{1}{7}\right)^1\left(\frac{6}{7}\right)^5 \\ &= 1 - \frac{46{,}656}{117{,}649} - \frac{46{,}656}{117{,}649} && \text{Simplify.} \\ &= \frac{24{,}337}{117{,}649} && \text{Subtract.} \end{aligned}$$

The probability that Martin will score at least 2 goals is $\frac{24{,}337}{117{,}649}$ or about 21%.

CHECK Your Progress

A basketball player has a free-throw percentage of 75% before the last game of the season. The player takes 5 free throws in the final game.

2A. What is the probability that he will make exactly two free throws?

2B. What is the probability that he will make at least two free throws?

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CHECK Your Understanding

Examples 1, 2 (pp. 730–731)

Find each probability if a coin is tossed 3 times.

1. P(exactly 2 heads) **2.** P(0 heads) **3.** P(at least 1 head)

Four cards are drawn from a standard deck of cards. Each card is replaced before the next one is drawn. Find each probability.

4. P(4 jacks) **5.** P(exactly 3 jacks) **6.** P(at most 1 jack)

SPORTS Lauren Wible of Bucknell University was the 2005 NCAA Division I women's softball batting leader with a batting average of .524. This means that the probability of her getting a hit in a given at-bat was 0.524.

7. Find the probability of her getting 4 hits in 4 at-bats.

8. Find the probability of her getting exactly 2 hits in 4 at-bats.

Exercises

HOMEWORK HELP

For Exercises	See Examples
9–30	1, 2

Find each probability if a coin is tossed 5 times.

9. P(5 tails) **10.** P(0 tails)

11. P(exactly 2 tails) **12.** P(exactly 1 tail)

13. P(at least 4 tails) **14.** P(at most 2 tails)

Find each probability if a die is rolled 4 times.

15. P(exactly one 3) **16.** P(exactly three 3s)

17. P(at most two 3s) **18.** P(at least three 3s)

Real-World Link

The word *Internet* was virtually unknown until the mid-1980s. By 1997, 19 million Americans were using the Internet. That number tripled in 1998 and passed 100 million in 1999.

Source: UCLA

As a maintenance manager, Jackie Thomas is responsible for managing the maintenance of an office building. When entering a room after hours, the probability that she selects the correct key on the first try is $\frac{1}{5}$. If she enters 6 rooms in an evening, find each probability.

19. P(never the correct key)
20. P(always the correct key)
21. P(correct exactly 4 times)
22. P(correct exactly 2 times)
23. P(no more than 2 times correct)
24. P(at least 4 times correct)

Prisana guesses at all 10 true/false questions on her history test. Find each probability.

25. P(exactly 6 correct)
26. P(exactly 4 correct)
27. P(at most half correct)
28. P(at least half correct)

29. CARS According to a recent survey, about 1 in 3 new cars is leased rather than bought. What is the probability that 3 of 7 randomly selected new cars are leased?

30. INTERNET In a recent year, it was estimated that 55% of U.S. adult Internet users had access to high-speed Internet connections at home or on the job. What is the probability that exactly 2 out of 5 randomly selected U.S. adults had access to high-speed Internet connections?

If a thumbtack is dropped, the probability of it landing point-up is 0.3. If 10 tacks are dropped, find each probability.

31. P(at least 8 points up)
32. P(at most 3 points up)

33. COINS A fair coin is tossed 6 times. Find the probability of each outcome.

Graphing Calculator

BINOMIAL DISTRIBUTION For Exercises 34 and 35, use the following information. You can use a TI-83/84 Plus graphing calculator to investigate the graph of a binomial distribution.

Step 1 Enter the number of trials in L1. Start with 10 trials.

KEYSTROKES: [STAT] 1 [▲] [2nd] [LIST] [▶] 5 [X,T,θ,n] [,] [X,T,θ,n] [,] 0 [,] 10 [)] [ENTER]

Step 2 Calculate the probability of success for each trial in L2.

KEYSTROKES: [▶] [▲] [2nd] [DISTR] 0 10 [,] .5 [,] [2nd] [L1] [)] [ENTER]

Step 3 Graph the histogram.

KEYSTROKES: [2nd] [STAT PLOT]

Use the arrow and [ENTER] keys to choose ON, the histogram, L1 as the Xlist, and L2 as the frequency. Use the window [0, 10] scl: 1 by [0, 0.5] scl: 0.1.

34. Replace the 10 in the keystrokes for steps 1 and 2 to graph the binomial distribution for several values of n less than or equal to 47. You may have to adjust your viewing window to see all of the histogram. Make sure Xscl is 1.

35. What type of distribution does the binomial distribution start to resemble as n increases?

EXTRA PRACTICE
See pages 919, 937.

Math Online
Self-Check Quiz at algebra2.com

H.O.T. Problems

36. OPEN ENDED Describe a situation for which the P(2 or more) can be found by using a binomial expansion.

37. **REASONING** Explain why each experiment is not a binomial experiment.

a. rolling a die and recording whether a 1, 2, 3, 4, 5, or 6 comes up

b. tossing a coin repeatedly until it comes up heads

c. removing marbles from a bag and recording whether each one is black or white, if no replacement occurs

38. **CHALLENGE** Find the probability of exactly m successes in n trials of a binomial experiment where the probability of success in a given trial is p.

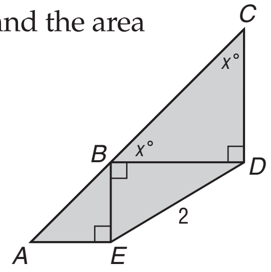
39. ***Writing in Math*** Use the information on page 735 to explain how you can determine whether guessing is worth it. Explain how to find the probability of getting any number of questions right on a 5-question multiple-choice quiz when guessing and the probability of each score.

STANDARDIZED TEST PRACTICE

40. **ACT/SAT** If $DE = 2$, what is the sum of the area of $\triangle ABE$ and the area of $\triangle BCD$?

A 2 C 4

B 3 D 5



41. **REVIEW** An examination consists of 10 questions. A student must answer only one of the first two questions and only six of the remaining ones. How many choices of questions does the student have?

F 112 H 44

G 56 J 30

Spiral Review

A set of 400 test scores is normally distributed with a mean of 75 and a standard deviation of 8. (Lesson 12-7)

42. What percent of the test scores lie between 67 and 83?

43. How many of the test scores are greater than 91?

44. What is the probability that a randomly-selected score is less than 67?

45. A salesperson had sales of \$11,000, \$15,000, \$11,000, \$16,000, \$12,000, and \$12,000 in the last six months. Which measure of central tendency would he be likely to use to represent these data when he talks with his supervisor? Explain. (Lesson 12-6)

Graph each inequality. (Lesson 2-7)

46. $x \geq -3$

47. $x + y \leq 4$

48. $y > |5x|$

GET READY for the Next Lesson

PREREQUISITE SKILL **Evaluate $2\sqrt{\frac{p(1-p)}{n}}$ for the given values of p and n. Round to the nearest thousandth if necessary.** (Lesson 5-2)

49. $p = 0.5, n = 100$

50. $p = 0.5, n = 400$

51. $p = 0.25, n = 500$

52. $p = 0.75, n = 1000$

53. $p = 0.3, n = 500$

54. $p = 0.6, n = 1000$